a Aurigæ. 291 lines have been measured in the spectrum of this star between λ 410 and λ 470, all of which appear identical with solar lines.

MAGNITUDE AND COLOUR OF η ARGÛS.—Observations of this variable have been made at Cordoba since 1871, and some comparisons made by Mr. Thome (*Astr. Nachr.*, No. 2922) show that it steadily decreased in magnitude until about the end of 1886, when a minimum of 7·65 was reached, and it is now about 6·6. In 1843, Maclear gave the brightness of η Argûs as − 1·0, or between that of Sirius and Canopus, so that the variation in magnitude is 8·5, and not 6 as heretofore assumed, this variation, extending over 44 years, gives an average yearly rate of diminution of 0·2.

It is interesting to note that the change in magnitude was accompanied by a change in colour; for although before minimum the star was of a dull scarlet the colour became lighter, until in June 1889 it was a bright orange.

ORBIT OF BARNARD'S COMET 1884 II.—From an investigation of all the available observations of this periodic comet, Dr. Berberich has computed the following elements (*Astr. Nachr.*, 2938–39).

Epoch 1884 August 16·5, Berlin Mean Time.

$M = 359^\circ\ 59'\ 49''\cdot13$
$\omega = 301\ \ 1\ \ 58\cdot63$
$\Omega = 5\ \ 8\ \ 59\cdot12$
$\iota = 5\ \ 27\ \ 38\cdot40$
$\phi = 35\ \ 44\ \ 50\cdot92$
$\mu = 657''\cdot0839 \pm 0''\cdot8876$
$\log a = 0\cdot4882572$
Perihelion passage = 1884 August 16·516543
Period = 1972·35 ± 2·66 days.

It will be seen from the foregoing period, that the comet will be at perihelion again in 1890 January 9·87.

ALGOL.—At the meeting of the Royal Prussian Academy of Sciences, held on November 28, Prof. Vogel gave the results he had obtained from photographs of the spectrum of this variable. Prof. Pickering had pointed out, some years ago, that if the variation in stars of the Algol class were due to the transit of a dark satellite across the disk of its primary, producing a partial eclipse, then since in every case yet known the two bodies must be close to each other, and of not very disproportionate size, the primary must revolve with very considerable rapidity in an orbit round the common centre of gravity of the two; and, therefore, be sometimes approaching the earth with great rapidity and sometimes receding from it. Six photographs of the spectrum of Algol—obtained, three during last winter, and three during the November just past—have shown that before the minimum the lines of the spectrum of Algol are markedly displaced towards the red, showing a motion of recession; but that after the minimum the displacement is towards the blue, showing a motion of approach. Assuming a circular orbit for the star, and combining the details given by the spectroscope with the known variation of the star's light, Prof. Vogel derives the following elements for the system of Algol:—

Diameter of Algol...	1,074,100 English miles.
Diameter of the dark companion	840,600 ,, ,,
Distance of centre...	3,269,000 ,, ,,
Speed of Algol in its orbit	27 miles per second.
Speed of the companion in its orbit ...	56 ,, ,,
Mass of Algol	$\frac{4}{9}$ of the sun.
Mass of the companion	$\frac{2}{9}$,, ,,
Speed of translation of the entire system towards the earth	2 miles per second.

It will be seen that the density both of Algol and its companion is much less than that of the sun—less than a quarter, in fact. This is what we might expect, for Algol and all the variables of its class yet examined give spectra of Group IV., and should therefore represent a less advanced stage of condensation than that seen in our sun. This demonstration of the truth of the satellite theory of variation of the Algol type derives also an especial interest from Prof. Darwin's researches on tidal evolution, for assuming, as we well may, that the cause of variation is the same in all members of the class, we now know of nine stars in which a large companion is revolving round its primary at but a very short distance from it, and in a very short space of time. The companion of U Ophiuchi must, indeed, be almost in contact with its parent star.

DISCOVERY OF A NEW COMET.—A faint comet was discovered by M. Borrelly at Marseilles, on December 12, at 7h. 49·5m. G.M.T. R.A. 18h. 7m.; daily motion in R.A. + 1m. 12s. N.P.D. 41° 7′; daily motion + 60′.

GEOGRAPHICAL NOTES.

WE regret to have to record the death of Major Peter Egerton Warburton, whose name will always be intimately associated with the history of exploration in Australia. He died at Beaumont, Adelaide, in his seventy-sixth year. His most famous achievement, undertaken in 1873, was the crossing of the continent from a point on the overland telegraphic line to the De Grey River, in Western Australia. Nothing was heard of him for about twelve months, during which he and his party suffered terrible privations in their march across the desert. After the expedition, Major Warburton visited England, and was awarded a Gold Medal of the Royal Geographical Society for his efforts towards increasing our knowledge of the interior of Australia. He received the Companionship of the Order of St. Michael and St. George in 1875.

THE death is announced of Cardinal G. Massaja in his eighty-first year, at St. Georgio a Cremano. For nearly half a century the name of this distinguished explorer has been intimately associated with the progress of geographical discoveries in Abyssinia and the surrounding regions. It was at his suggestion that the Italian Geographical Society organized the Antinori Expedition to Shoa, which has resulted in the occupation of a vast region, and the extension of Italian influence over the whole of Ethiopia. His chief work, "I miei trentacinque Anni nell' alta Etiopia," abounds in valuable geographical, historical, and ethnological information on the East African regions for so many years explored and studied by him. The Cardinal was born at Piovà in 1809, and, in 1846, appointed Vicar Apostolic of the Galla nation.

FROM the Berlin Correspondent of the *Daily News* we learn that a full account of the ascent of Kilimanjaro by Dr. Hans Meyer and Prof. Purtscheller has been received at Berlin. It is dated "Marangu Jagga, October 9." The journey from Zanzibar to Uawela took exactly a fortnight. On September 25 the travellers reached Marangu. On October 2 they encamped, with a Pangani negro, on the ridge of the plateau, at a height of 14,450 feet. At 2.30 a.m. they started for the lava-ribs surrounding the valley of glaciers to the south about 1200 feet above. At 7 o'clock, on the right side of the valley, at an elevation of about 16,500 feet, the first snow was seen under cover of the rocks. The higher they went, the more clefts and fissures the field of ice had. The travellers say:—"After great exertions we reached, at 1.45, the snow-line, and it was seen that the highest peak, which was formed of rocks jutting out of the snow, was about one and a half hour's march to the left. After resting a day and a half we set off, on October 5, to bivouac in the Lava Cave, at a height of about 15,200 feet, and on the next day we repeated the ascent. The peaks were gained without particular difficulty, and on the central and highest one, 19,680 feet above the sea, the German flag was planted." Dr. Meyer proposes to call this peak Kaiser Wilhelm Peak. The view from here on to the Kibbs Crater—which is 6600 feet broad and 660 feet high, and the lower half of which is encased in a mighty belt of ice, whilst a volcanic cone of about 500 feet rises in the centre—is magnificent. The beauties of the landscape in the Kilimanjaro region seem to be quite extraordinary. On October 10 the Kimawensi was to be ascended. The two travellers enjoy the best of health.

THE double number of the *Bollettino* of the Italian Geographical Society for October and November, which appears some weeks behind time, is largely devoted to African subjects, and more particularly to the north-eastern region, which is rapidly becoming an "Italian colony." Captain D. Stasio publishes a summary of Don Francesco Alvarez's "Travels in Ethiopia" in the sixteenth century, enriched with valuable notes and additions. Alvarez, a priest attached to an embassy forwarded by Portugal, in 1520, to the Emperor of Abyssinia, shows himself a careful observer of men and things, and his work, which was included in Ramusio's "Navigationi et Viaggi" (Venice, 1588), abounds in details regarding the political, social, and economic relations of that region in the sixteenth century. Giulio D. Cocorda brings to a conclusion his important series of papers

© 1889 Nature Publishing Group

on the South African gold-fields, which include much information on the present condition of the whole of South Africa as far north as the Zambesi. The observer points out that, while the Delagoa Bay and other lines of communication are much discussed, the fine artery of the perfectly navigable Limpopo is entirely neglected, notwithstanding Captain Chaddock's navigation of it a few years ago. The writer remarks that "this river flows mainly through regions under the influence or protectorate of England; the Transvaal people on the one side, and those of Matabeleland on the other, would certainly be glad to avail themselves of this outlet for their produce. As it traverses only a small tract of Portuguese territory about its estuary, I hope and believe that Portugal will not be allowed to treat the Limpopo as she is now attempting to treat the Zambesi. The subject is of such importance that it cannot fail soon to be brought before the British Parliament." Referring to the negotiations at present going on in connection with the Swaziland question, he observes, in the same spirit:—"The Swazi people must, sooner or later, yield either to the Transvaal or to England, and if to the former, it must be to the entire detriment of British interests. England, as the suzerain power in South Africa, should be the first in the field, both in her own interest and in that of her other colonies and subjects. If she does not assume the protectorate of Swaziland, besides losing the control of a vast and rich mineral district, she will deprive the colony of Natal of all further hope of expansion. If she ignores her responsibility in this matter, and allows the Transvaal Republic to absorb Swaziland, she will add another to the long list of blunders that threaten to destroy all prospect of consolidating a dominion as large as Canada, and may end disastrously for British interests in South Africa."

A FRENCH traveller has just achieved a feat of great interest. Captain Trivier, equipped by the newspaper *La Gironde*, started some eighteen months ago for the Congo State. He went up the river to Stanley Falls, and thence proceeded to Central Africa and the Lake region, accompanying caravans. He has just arrived at Mozambique.

Globus reports that during the past summer M. Thoroddsen, the well known student of Iceland, has carried out a journey in the waste region known as Fiskivötn, lying between Hecla and the Vatna Jökul, which has hitherto been unvisited for the most part by any inquirer. To the east and north of Hecla he discovered a new obsidian region. Crossing the Tunguaa, he went to the Fiskivötn group of lakes, all true crater lakes. The district between this and the Vatna Jökul has absolutely no plant-life whatever; it consists of lava-fields, and plains of volcanic sand. In it he found a lake, Thorisvatn, the second largest in the island. Thence, after a day's journey through an utterly desolate district, he reached the hitherto unknown source of the Tunguaa. To the south of this he discovered, between three ranges of hills, previously unknown, a new and very long lake.

MR. DAUVERGNE has, says the *Times of India*, completed an adventurous journey in the regions of North-West Cashmere. His course was from Leh northwards to the Kilian Pass, in Kashgaria, and then northwards across the Pamir to the Upper Oxus. He reached Sarhad in safety, and after six days' halt there, crossed the Hindu Kush by the Baroghil Pass, as he did not wish to visit Chitral. He then turned eastwards, and after a trying journey through the snow, crossed the Ishkaman Pass, north of Yasin. Thence he travelled southwards by the Karambar Valley, and eventually reached Gilgit, a short time after Captain Durand had started for Chitral. Mr. Dauvergne reports that the Russian explorer, Captain Grombchevsky, whose attempt to reach Kafiristan was noticed some time ago, was stopped at Kila Panjah on the Oxus, by the Afghan authorities.

THE ST. PETERSBURG PROBLEM.

THIS celebrated problem, which is first mentioned before 1708 in a letter from the younger Nicholas Bernoulli to Montmort, has been frequently discussed by Daniel Bernoulli (1730) and other eminent mathematicians. It may be briefly stated as follows:—

A tosses a coin, and undertakes to pay B a florin if head comes up at the first throw, two florins if it comes up at the second, four florins if it be deferred until the third throw, and so on. What is the value of B's expectation?

The chance of head appearing at the
1st, 2nd, 3rd, 4th . . . . *n*th throw is
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots \ldots \frac{1}{2}^n$. A promises to pay for head
$1, 2, 4, 8 \ldots \ldots 2^{n-1}$ florins, hence B's expectation is
$\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16} \ldots \ldots 2^{n-1}/2^n = \frac{1}{2}$ florin.

Hence the total value of B's expectation is an *infinite* series, each term of which is a shilling, or it is infinite.

This result of the theory of probability is apparently directly opposed to the dictates of common-sense, since it is supposed that no one would give even a large finite sum, such as £50, for the prospect above defined.

Almost all mathematical writers on probability have allowed the force of the objection, which they have endeavoured to evade by various ingenious artifices all more or less unsatisfactory.

The real difficulty of the problem seems to lie in the exact meaning of *infinite* and *value of the expectation.*

Since the infinite value of the result is *only* true if an infinite number of trials are paid for and made, all such considerations as want of time and the bankruptcy of A or B are precluded by the terms of the question.

The value of B's expectation is frequently confused with how much he can or ought to pay for it; thus Mr. Whitworth ("Choice and Chance," p. 234) finds that if B have 1024 florins, he may give very little more than 6 florins for the venture. This ingenious, solution seems to have no reference to the original problem, which has been modified by Mr. Whitworth's introduction of the word "advantageously" (p. 232).

B can pay for his expectation in three ways: (i.) a sum before each toss; (ii.) a sum before each series of tosses ending with head; (iii.) a sum for the total result of A's operations.

Mr. Whitworth apparently assumes the first method of payment, and shows that the larger B's funds are the more he may safely pay for each toss, since he can continue to play longer. Many mathematicians take the second method of payment. "However large a fee I pay for each of these sets, I shall be sure to make it up in time" ("Logic of Chance," p. 155).

It is easy to show in this case also that what may be safely paid before each series increases with the number of series.

Suppose a very large number of tosses made, about half would come up heads and half tails; each head would end a series, when a fresh payment must be made by B. Suppose the tosses limited to one series, if B pays one florin he cannot possibly lose, if he pay anything more he may lose by head coming up the first time, and the more he pays the greater will his chance of loss be, since the series of tails must be longer to cover it. But, however large a finite sum he pays, he is not *certain* to lose, *e.g.* head *may* not come up till the hundred and first toss, when he would receive

2^{100} = 1,267 650,600 228,229 401,496 703,205 376 florins.

If the sets are limited to one hundred, about

50 heads would probably come up the	1st toss.	B would receive for each series 50 florins.
25 ,, ,, ,,	2nd ,,	
13 ,, ,, ,,	3rd ,,	
6 ,, ,, ,,	4th ,,	
3 ,, ,, ,,	5th ,,	
2 ,, ,, ,,	6th ,,	
1 ,, ,, ,,	7th ,,	

Hence for the hundred sets, B would receive about 350 florins, or he could pay without loss seven shillings for each set.

If N be the number of sets, the total amount received by B will probably not be less than n terms of the series

$$\left\{\frac{N \times 2^0}{2^1} + \frac{N \times 2^1}{2^2} + \&c.\right\} = n\{\tfrac{1}{2}\}N,$$

but n is the number of times which N is successively divisible by 2, or $2^n = N$, or $n = \log N/\log 2$. But the amount x which B can afford to pay per set when multiplied by the number of sets is equal to the amount which he receives, or—

$$xN = \frac{\log N}{\log 2}\{\tfrac{1}{2}\}N,$$

hence $x = \log N/0{\cdot}6$ nearly.

This formula, though inexact for low, is very convenient for high, values of N.

$N = 1$	$x = 0$	$N = 10^6$	$x = 10$
$= 50$	$= 2{\cdot}7$	$= 10^9$	$= 15$
$= 100$	$= 3{\cdot}3$	$= 10^{12}$	$= 20$
$= 1000$	$= 5$	$= 10^{15}$	$= 25$

x increases with, though much more slowly than, N, and becomes infinite when N does. But to justify a payment of

© 1889 Nature Publishing Group